MATHEMATICAL MODELING OF INTERNAL GRAVITY WAVES (IGW) DYNAMICS IN OCEAN

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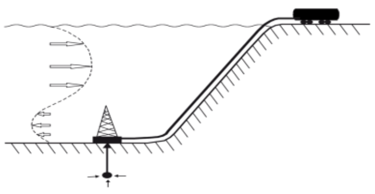
«Fram» by F.Nancen in Arctic ocean:
«Dead Water» phenomena

The history of studying the internal gravity waves (IGW) in the ocean, as is known, originated in the Arctic Region after F. Nansen had described the phenomenon called ≪Dead Water≫. Nansen was the first to observe internal gravity waves in the Arctic ocean. The notion of internal waves involves different oceanic phenomena, such as: ≪Dead Water≫, internal tidal waves, large scale oceanic circulation, and powerful pulsating internal waves. Such natural phenomena exist in the atmosphere as well; however, the theory of internal waves in the atmosphere was developed later along with the progress of the aircraft industry and aviation technology. Studying the oceanic currents of the Arctic ocean became the principal objective of the «Fram» expedition in 1893–1986 and was continued in the years to follow. During the expedition the scientists made a lot of observations and collected many data sheets and measurements in the Arctic which had been essentially unexplored at the time. F. Nansen was the first scientist to classify the manner in which the ≪Dead Water≫ phenomenon occurs. This phenomenon comes about from internal gravity waves generated by a slow moving vessel. The first theoretical work dedicated to internal gravity waves was the thesis by V.W. Ekman, who provided a detailed definition of dead water and systematized the data obtained by F. Nansen. The ≪Dead Water≫ effect from IGW has been long known to sailors. Sailing vessels after being caught in the thermocline (a density contrast layer) were suddenly brought down to a complete stop. This phenomenon resulted from the IGW generated by a vessel. But since then sailors saw no waves on the surface behind the ship this enormous water resistance seemed to be inexplicable whatsoever, and they blamed the bewitched drowned for holding the ship in place and not letting her go.

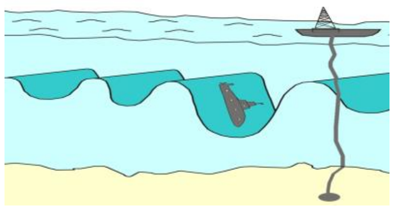
Up to the 1960-s the research was for most part focused on tidal waves, however, in the middle of the 1950-s some theoretic developmental studies and laboratory investigations were undertaken that involved internal pulsating waves. As early as in 1950 there appeared the first definition for a superficial wake of IGW in the ocean. In 1965 first scientific observations were made concerning the oceanic large amplitude internal waves and solitons . The interest to investigations involving IGW grew up after the WW2 when the US Navy lost a few of its most advanced at the time submarines. After those accidents there were assumptions made that the disaster might have been caused by IGW. The submarines often move along the thermocline to avoid detection since the thermocline surface reflects acoustical signals of active sonars and sea vessels. The most notorious incident involved the US Navy Thresher submarine that was lost at sea in 1963 with the crew of 129 on board. This submarine was a most advanced boat in the world in the 1960-s and she could descend to depths and move at velocities that were inconceivable just a few years before she was constructed. It might be that the Thresher submarine was going along the thermocline and a large IGW took her down to a depth pressure that she could not survive.

Modern methods of IGW investigations in ocean –asymptotical methods. Journal of Engineering Mathematics (2011). "…Applied mathematics seems, at first sight anyway, to become more and more dominated by direct numerical simulations. Admittedly, this leads to new insights which, it would seem, could not have been attained by other means and many believe that asymptotics deals with exceptional cases which are usually outside the practical domain. However, this is a misconception!…Does the asymptotic analysis have a future in the era of supercomputers? The answer to the question is obvious: Yes, it has…Why do we need this complicated analysis, if the problem can be easily solved by computer simulations?..Before the 90s numerical calculations were complementary to rigorous or approximate analysis of a problem. At the present time the "center of mass" in research is still moving towards computations. However, along this way, it is getting more and more clear that computer simulations being applied to practical problems is not a universal panacea. Euphoria about computers is still high but a more practical point of view is becoming at least visible these days: computation is a research tool as are many other approximate or rigorous tools of mathematics which help us to gain new knowledge, understand important phenomena and solve challenging practical problems…"

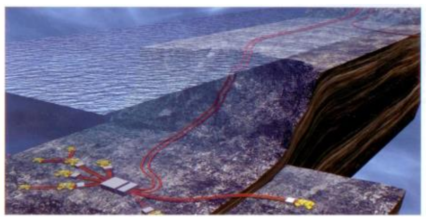
Applications. IGW in the ocean have been studied for a long time: on this subject have been published many papers in the years 1960-1980. Recently IGW studies have not attracted much attention: significantly decreased the number of IGW theory publications At the present time: appearance of new directions in IGW research: IGW fields can generate short-live intensive waves with very large amplitude – "Killer waves" : properties of these IGW waves is similar to the properties of surface "Killer waves" ; IGW and shear flows can deform pillars of oil platforms: this deformation was recently discovered in the oil-gas deposit; monitoring IGW system is similar to a tsunami monitoring system; deep ocean: IGW can cause significant movement of material near the bottom the effect of surface waves at the ocean bottom is not significant; classical problems of the IGW influence on the ocean surface remain actual.



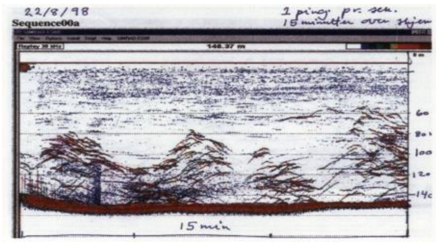
Continental shelf industrial activities and extraction of oil-gas are important stimulus for IGW investigations. Oil-gas deposit: ship and sea platforms use a very long pipelines. IGW effect on underwater objects IGW calculations allow to determine the underwater equipment malfunction



The construction of underwater objects is necessary to constantly measure the IGW and flows:
solution of IGW dynamics fundamental problem makes it possible not to do expensive IGW measurements in ocean
Special interest of IGW investigation – safety of various underwater objects (submarine, sea platforms)



Oil-gas deposit Ormen Lange(Norway)
Pipeline 100-150 km - strong influence of IGW over slope



Ocean ecology problems
A lot of fish marks IGW packet (Barents sea)
IGW packets transfer biomass, plancton, nutrients, fish

Main fundamental problems : i) mathematical modeling of IGW dynamics in horizontally non-uniform and non-stationary stratified ocean; ii) numerical modeling of IGW dynamics from non-local sources :sea platform and construction, underwater vessels and others; iii) asymptotic methods development for investigation of IGW dynamics in horizontally non-uniform and non-stationary stratified ocean; iv) creation of algorithms for processing the IGW measurements in ocean

Basic model. Equations of motion

$$\rho\frac{dW}{dt}+\frac{\partial p}{\partial z}+g\rho=0 \quad \rho\frac{dU_1}{dt}+\frac{\partial p}{\partial x}=0 \quad \rho\frac{dU_2}{dt}+\frac{\partial p}{\partial y}=0$$

Equation of state (adiabatic approximation, c- sound velocity)

$$\frac{1}{c^2}\frac{dp}{dt}=\frac{d\rho}{dt} \qquad \frac{d}{dt}=\frac{\partial}{\partial t}+U_1\frac{\partial}{\partial x}+U_2\frac{\partial}{\partial y}+W\frac{\partial}{\partial z}$$

Equation of continuity

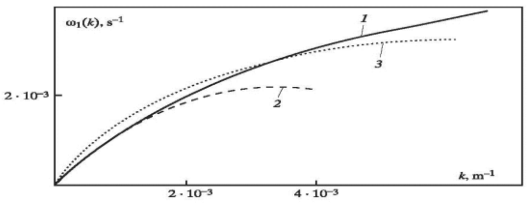
$$\frac{\partial\rho}{\partial t}+div(\rho\mathbf{U})=0$$

Boundary condition: non-flow condition on bottom, free/surface/ «rigid cover» on surface $\mathbf{U}=(U_1,U_2,W)$ - vector of velocity, $p(\rho)$ - pressure (density)

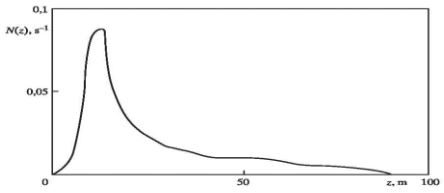
Main IGW vertical spectral problem

$$\frac{\partial^2\varphi_n(z,k)}{\partial z^2}+k^2\left(\frac{N^2(z)}{\omega_n^2(k)}-1\right)\varphi_n(z,k)=0$$

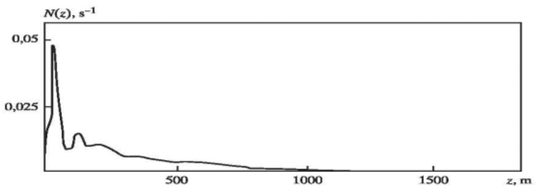
Brent-Vaisaila frequency (buoyancy frequency) *N(z)*-main parameter of IGW dynamics in ocean.



Far IGW fields in ocean depend on behavior of dispersion relation at small wave number. Dispersion curve and it's approximation at small wave number *k=0*



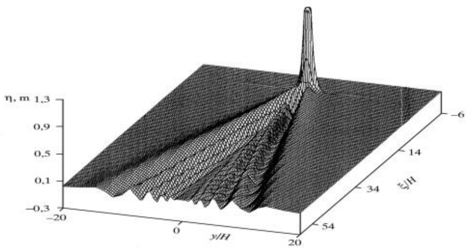
Brent-Vaisaila frequency (buoyancy frequency) *N(z)* distribution in sea



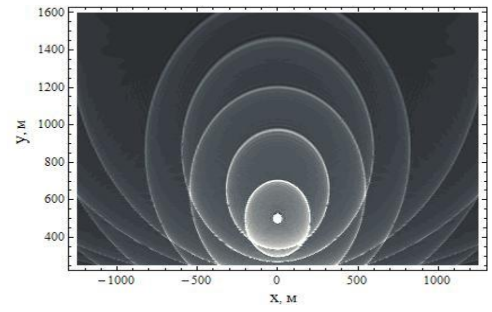
Brent-Vaisaila frequency (buoyancy frequency) *N(z)* distribution in ocean

Real ocean. Exact analytical solutions for real ocean parameters exist if density distribution and bottom topography – analytical model function: only numerical and asymptotic (WKBJ method) solutions are possible

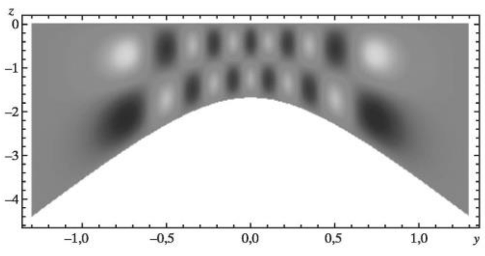
$$\mathbf{V}(z,\varepsilon x,\varepsilon y,\varepsilon t)=\sum_{m=0}^{\infty}(i\varepsilon)^m\mathbf{V}_m(z,\varepsilon x,\varepsilon y,\varepsilon t)\exp\left(\frac{iS(x,y,t)}{i\varepsilon}\right) \qquad \mathbf{V}(z,\varepsilon x,\varepsilon y,\varepsilon t)=(U_1(z,\varepsilon x,\varepsilon y,\varepsilon t),U_2(z,\varepsilon x,\varepsilon y,\varepsilon t),W(z,\varepsilon x,\varepsilon y,\varepsilon t))$$



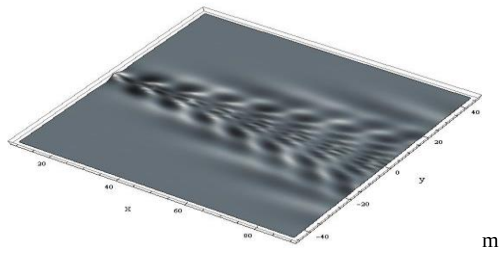
IGW from moving source in ocean



IGW structure in ocean with non-uniform depth



IGW propagation over underwater hill



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IGW far fields from pulsating sources

Main results. 3D model of IGW generation by sources in ocean is constructed, the solution of the problem is expressed in terms of the Green's function and the asymptotic representations of the solutions are obtained. Uniform asymptotic forms of IGW in horizontally inhomogeneous and non-stationary stratified ocean are obtained. Modified WKBJ method for IGW dynamics investigation in ocean is developed; - ocean bottom topography and stratified ocean medium structure determine main parameters of IGW fields. IGW dynamics are strongly influenced by non-stationarity and horizontal inhomogeneity in ocean. Effect of space-frequency blocking of IGW is obtained: IGW fields localized in limited spatial domain (captured waves), IGW fields propagate over long distances (progressive waves). Spatial domain where progressive waves propagate is depend on: density distribution and variability, bottom topography. Obtained asymptotic is suitable for detection of IGW fields by aerospace radiolocation methods.

Publications (Monographs) 1) В.В.Булатов, Ю.В.Владимиров Внутренние гравитационные волны в неоднородных средах. М.: Наука, 2005, 195 с.(in Russian); 2) V.V.Bulatov, Y.V.Vladimirov Internal gravity waves: theory and applications. Moscow, Nauka Publishers, 2007, 304 pp.; 3) В.В.Булатов, Ю.В.Владимиров Динамика негармонических волновых пакетов в стратифицированных средах. М.: Наука, 2010, 470 с.(in Russian); 4)V.V.Bulatov, Y.V.Vladimirov Wave dynamics of stratified medium. Moscow, Nauka Publishers, 2012, 584 pp; 5) В.В.Булатов, Ю.В.Владимиров Волны в стратифицированных средах. М.: Наука, 2015, 735 с.(in Russian).