DYNAMICS OF NON-HARMONIC INTERNAL GRAVITY WAVES IN STRATIFIED INHOMOGENEOUS MEDIUMS

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ABSTRACT
The uniform asymptotic form of the internal gravity waves field generated by a source moving above the smoothly varying bottom is constructed. The problem of reconstructing non-harmonic internal gravity wave packets generated by a source moving in a stratified ocean is considered. The solution is proposed in terms of wave modes, propagating independently at the adiabatic approximation, and described as a non-integral degree series of a small parameter characterizing the stratified medium. A specific form of the wave packets, which can be parameterized in terms of model functions (Airy functions), depends on a local behavior of the dispersion curves of individual wave mode. A modified space-time ray method was proposed, which belongs to the class of geometrical optics methods. The key point of the proposed technique is the possibility to derive the asymptotic representation of the solution in terms of a non-integral degree series of the some small parameter.

INTRODUCTION

In this study we analyze the evolution of non-harmonic wave packets of internal gravity waves generated by a moving source under the assumption that the parameters of a vertically stratified medium (e.g. an ocean) vary slowly in the horizontal direction and in time, as compared to the characteristic length of the density $\rho(x,y,z)$. Specifically, we assume that a point source is moving with the supercritical velocity V along the x-axis at the depth z_0 in a stratified layer $-H(\varepsilon x,\varepsilon y)<z<0$ (ε is a small parameter) of the stratified fluid with the Brunt-Vaisala frequency $N(z)$. The wave dynamics is defined by [1,2]

$$\frac{\partial^2}{\partial t^2}\left(\Delta+\frac{\partial^2}{\partial z^2}\right)W+N^2(z)\Delta W=\delta_{tt}''(x+Vt)\delta(y)\delta'(z-z_0)$$

$$\Delta \mathrm{U}+\nabla\frac{\partial W}{\partial z}=\delta(z-z_0)\nabla(\delta(x+Vt)\delta(y)),$$

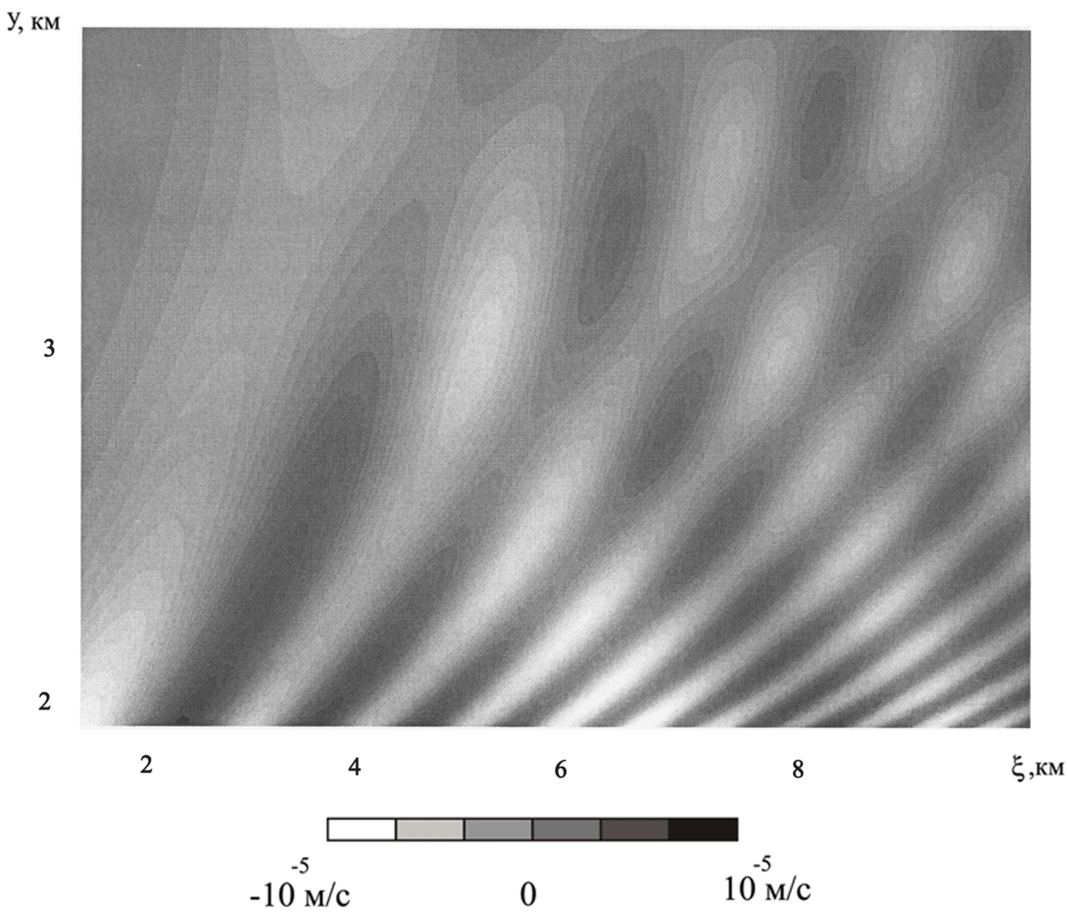
where $\Delta=\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}$ $\nabla=\left(\frac{\partial}{\partial x},\frac{\partial}{\partial y}\right)$, $\mathrm{U}=(\mathrm{U}_1,\mathrm{U}_2)$ and W are the horizontal and vertical velocities, respectively. A specific form of the wave packets, which can be parameterized in terms of model functions, e.g. Airy functions or Fresnel functions, depends on a local behavior of the dispersion curves of individual modes in the vicinity of corresponding critical points [1].

METHODS

We modified the space-time ray method, which belongs to the class of geometrical optics methods [1,3]. The key point of the proposed technique is the possibility to derive the asymptotic representation of the solution in terms of a non-integral degree series of the small parameter $\varepsilon=\lambda/L$, where λ is the characteristic wave length, and L is the characteristic scale of the horizontal heterogeneity. Specifically, we are looking for a solution as the sum of modes propagating independently (the adiabatic approximation), namely [1, 3]: $W=A(\varepsilon x,\varepsilon y,z,\varepsilon t)R_0(\sigma)+\varepsilon^a B(\varepsilon x,\varepsilon y,z,\varepsilon t)R_1(\sigma)+\ldots$, $\mathrm{U}=\mathrm{U}_0(\varepsilon x,\varepsilon y,z,\varepsilon t)R_1(\sigma)+\ldots$, $R_{i+1}'(\sigma)=R_i(\sigma)$ $\sigma\equiv\left(S(\varepsilon x,\varepsilon y,\varepsilon t)/a\varepsilon\right)^a$, where σ is on the order of one, and the functions $S(\varepsilon x,\varepsilon y,\varepsilon t)$, $A(\varepsilon x,\varepsilon y,z,\varepsilon t)$ and $\mathrm{U}_0(\varepsilon x,\varepsilon y,z,\varepsilon t)$ are to be found [1,3]. Depending on the presence of a uniform (non-stratified) sublayer, the function $R_0(\sigma)$ is expressed in terms of Airy functions (Airy wave in shelf zone) or the Fresnel integrals (Fresnel wave in deep ocean).

The explicit form of the asymptotic solution was determined based on the principles of locality and asymptotic behavior of the solution in case of a stationary and horizontally homogeneous medium. First, $\mathrm{U}_0(\varepsilon x, \varepsilon y, z, \varepsilon t)$ can be estimated with the $\varepsilon^{3/2}$-order of accuracy as: $\mathrm{U}_0 = -\frac{\partial \mathrm{A}}{\partial \mathrm{z}} \sqrt{2S} \nabla S \left| \nabla S \right|^{-2} \varepsilon^{-1} + O(\varepsilon^{3/2})$. The wave packet phase is calculated from the corresponding eikonal equations that are numerically solved along the characteristic curves. Specifically, the eikonal equation is defined as: $\frac{\partial^2 A}{\partial z^2} + |\mathrm{k}|^2 \left(\frac{N^2(z)}{\omega^2} - 1 \right) A = 0$, $\mathrm{k}(\omega, x, y) = -\nabla S \quad \omega = \frac{\partial S}{\partial t}, \left| \nabla S \right|^2 = |\mathrm{k}|^2$. The wave packet amplitudes are determined from the energy conservation laws along the characteristic curves. We represent $\mathrm{A}(\mathrm{x}, \mathrm{y}, \mathrm{z}, \mathrm{t})$ in the form $A(x, y, z, t) = \Psi(x, y, \omega(x, y, t)) \, f(x, y, z, \omega(x, y, t))$ where the function f is a normalized eigenfunction for the vertical spectral problem [1]. Then, as can be shown, the function $\Psi(x, y, \omega(x, y, t))$ is determined from the energy conservation law: $\frac{d}{dt} \ln \left(D \Psi^2 \frac{\partial K}{\partial \omega} K^{-1} \right) = 0$, where $K(x, y, \omega) = |\mathrm{k}|^2$ and D is the Jacobian determinant to define transformation from the ray coordinates into the Cartesian ones.

APPLICATIONS



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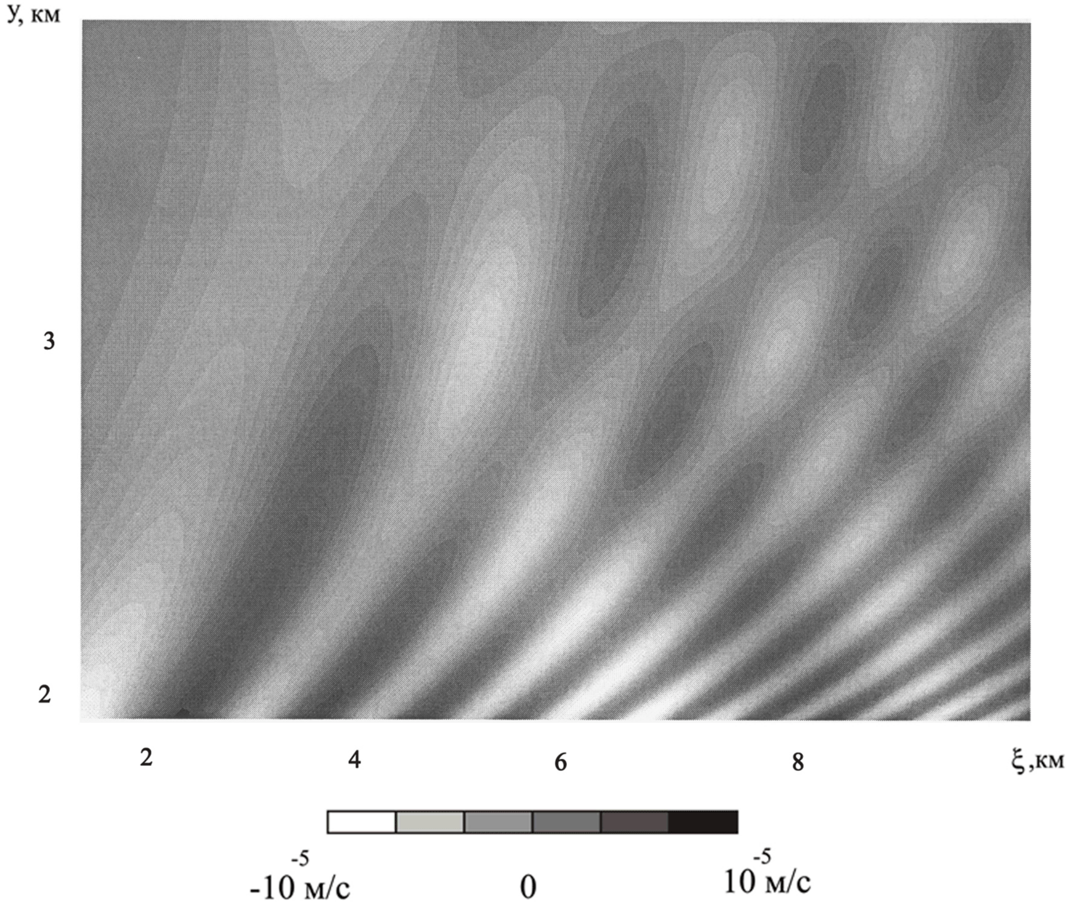

Fig. 1 Vertical component w of velocity in ($\xi = x + Vt, y$)) plane from a source moving in stratified media of variable depth.

In Fig.1 we present results of computation of the vertical velocity amplitude-phase picture. The main parameters of the computations are typical for a real oceanic shelf: the slope of the bottom10^0, $V = 2m/s$. It follows from the numerical results thus presented, outside the caustic, the wave field is sufficiently small indeed and is not subjected to great many oscillations, whereas the wave picture inside the zone of caustic is a rather complicated system of incident and reflected harmonics.

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